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LETTER TO THE EDITOR

Self-organized criticality in 1D traffic flow model with inflow or outflow

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Abstract. The asymmetric simple-exclusion model with parallel dynamics (the deterministic cellular automaton 184) is extended to take into account the exchange of cars between different lanes on a multi-lane roadway. The traffic flow model is presented by the one-dimensional asymmetric exclusion model with injection or extraction of particles. The system is driven asymptotically into a steady state exhibiting a self-organized criticality. The typical interval $\langle s \rangle$ between consecutive jams scales as $\langle s \rangle \approx L^{\nu}$ with $\nu = 0.62 \pm 0.04$ where L is the system size. It is shown that the jam-interval distribution $n_s(L)$ satisfies the finite-size scaling form $n_s(L) \approx L^{-\beta} f(s/L^{\nu})$ with $\beta = 2\nu$.

Recently, traffic problems have attracted considerable attention [1–5]. Cellular automaton (CA) models are being applied successfully to simulations of traffic. The one-dimensional (1D) asymmetric simple-exclusion model can be formulated into traffic flow problems. The 1D exclusion model is one of the simplest examples of a driven system [6, 7]. The model has been extensively studied to understand systems of interacting particles [8, 9]. The 1D exclusion model has been used to study the microscopic structure of shocks [10, 11] and is closely linked to growth processes [12–14]. The two-dimensional versions of the asymmetric simple-exclusion model have been applied to the traffic-jam problem in an entire city [2–5, 15].

Nagel and Schreckenberg [1] extended the 1D asymmetric exclusion model to take into account car velocity in order to simulate freeway traffic. They showed that a transition from laminar traffic flow to start–stop waves occurs with increasing car density, as is observed in actual freeway traffic. Musha and Higuchi [16] found that traffic flow on a highway shows a $1/f$ power spectrum, by directly measuring the traffic flow on an actual highway. Nagatani [17] studied the clustering of traffic in the extended asymmetric exclusion model taking into account the difference between the inherent velocities of individual cars. Ben-Naim *et al* [18] analysed the clustering of cars in a simple aggregation model.

Very recently, Nagel and Herrmann [19] showed that the open boundary version of the model exhibits a self-organized criticality, providing enough input and output of mass at the boundaries. The concept of self-organized criticality attracted considerable attention [20, 21]. Furthermore, Nagel [22] studied the lifetimes of simulated traffic jams for freeway traffic and found emergent traffic jams with a self-similar appearance. However, the CA model is not simple since it is described by the CA rule of the seven states and the scaling behaviour depending upon the system size is unclear.

In this letter, we present a 1D traffic flow model with inflow or outflow of cars. The CA model is an extended version of the 1D asymmetric simple-exclusion model to take into

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account injection or extraction of particles. We investigate a self-organized criticality in the system. The CA model mimics the traffic flow on a multi-lane roadway. The exchange of cars occurs between different lanes on the multi-lane roadway. The number of cars on a single lane fluctuates by inflow from other lanes or outflow to other lanes. The traffic flow on a single lane of the multi-lane roadway is described by the 1D asymmetric simple-exclusion model with injection or extraction of particles. The 1D asymmetric simple-exclusion model with parallel dynamics is consistent with the deterministic CA 184 [2]. Introducing injection or extraction of particles into the simple CA model drives the system asymptotically into a steady state exhibiting a self-organized criticality.

Our CA model is defined on a 1D lattice of L sites with periodic boundary conditions. Each site is occupied by one car or it is empty. For an arbitrary configuration, one update of the system consists of two steps. The first step is performed in parallel for all the cars. The movement or halting of cars in the first step is the same as the 1D asymmetric simple-exclusion model with parallel dynamics. Each car moves ahead one step unless the forward nearest-neighbour site is occupied by another car. If a car is blocked ahead by another car, it does not move even if the blocking car moves out of the site during the same time step. Then the second step is performed. In the second step, a site is selected randomly. If its site is unoccupied by one car, a car is injected on its site. When its site is occupied by one car, the car is extracted from its site.

We have performed simulations with the CA model starting with an ensemble of random initial conditions where the system size is $L = 250$–25000 and the initial density of cars is $p_0 = 0.0$–1.0. Each run is calculated up to 10^3–10^5 time steps. The data are averaged over 100 runs. For illustration, figure 1 shows a typical pattern of cars for the initial car density $p_0 = 0.2$ up to 500 time steps, where the system size is $L = 300$. The vertical direction indicates that in which cars move ahead. The horizontal direction is that of time. A car is indicated by a dot. The trajectory of a car is indicated by a curve. The region of grey colour represents that in which cars move with an interval of two sites. The local density of cars is $p = 0.5$. Cars within the region move with the maximal velocity 1. The region of black colour represents the appearance of a traffic jam in which cars are stopped because they are blocked by other cars. The traffic jam propagates backward. There are traffic jams with various life times. We study the distribution of the interval between traffic jams in a steady state after sufficiently many time steps. We investigate whether or not the system is driven asymptotically to a steady state with a self-organized criticality, by starting from any initial density p_0.

Figure 2 shows the time evolution of the car density p until $t = 500$ for the initial density $p_0 = 0.1$, 0.3, 0.7 and 0.9, where $L = 300$. For any initial density p_0, the car density p approaches a steady state of about $p = 0.5$. The density p fluctuates about the critical value $p_c = 0.5$. The steady-state concentration can be calculated analytically in the mean-field approximation. A particle is injected by the probability $(L - N)/L$ and is extracted by the probability N/L where N is the number of particles and L is the system size. The rate equation of particle number is described by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \left(\frac{L-N}{L}\right) - \frac{N}{L}. \tag{1}$$

The number N of particles is given by

$$N = \frac{L}{2} - \frac{(L-2N_0)}{2}\mathrm{e}^{-2t/L} \tag{2}$$

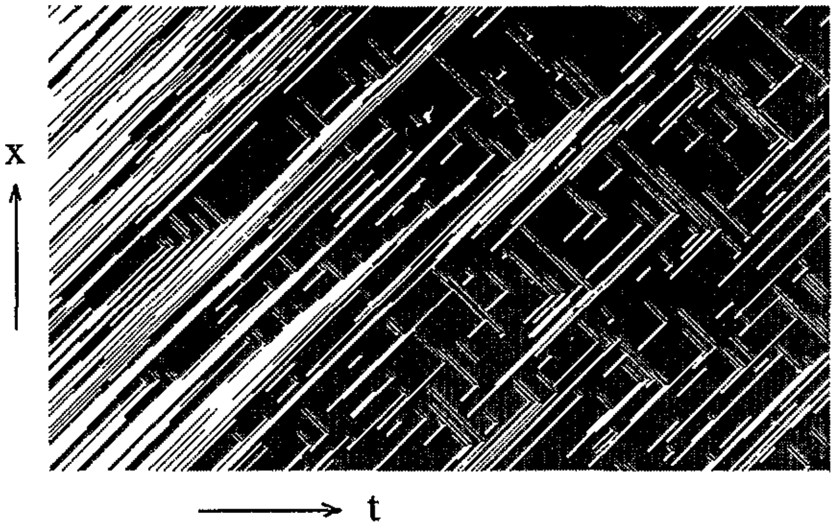

Figure 1. The typical configuration of cars up to 500 time steps for the initial car density $p_0 = 0.2$, where the system size is $L = 300$. The vertical and horizontal directions indicate respectively those of space and time. A car is indicated by a dot. The trajectory of a car is represented by a curve. The region of grey colour represents that in which cars move with the interval of two sites. The region of black colour represents the appearance of a traffic jam in which cars are stopped by the blocking of other cars.

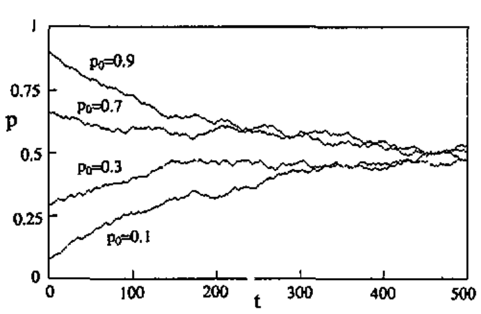

Figure 2. The time evolution of the car density p up to $t = 500$ for the initial densities $p_0 = 0.1$, 0.3, 0.7 and 0.9 where $L = 300$. For any initial density p_0, the car density p approaches a steady state of about $p = 0.5$.

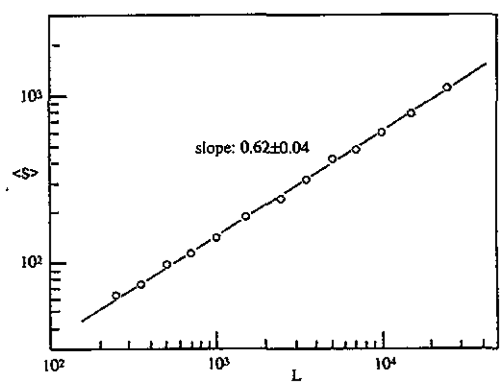

Figure 3. The log–log plot of the typical interval $\langle s \rangle$ of traffic jams against the system size L. The interval $\langle s \rangle$ scales as $\langle s \rangle \approx L^{0.62\pm0.04}$.

where N_0 is the particle number at the initial state. In the limit of $t \to \infty$, the density $\rho(= N/L)$ approaches the critical value 0.5.

There is a critical parameter in the model, namely the ratio R of the probabilities of putting in and taking out a car. $R = 1$ is the critical value, leading to the density $\rho = 0.5$ which is a very special density of the asymmetric exclusion model with parallel update and without randomness. The fundamental diagram for the density and velocity has a non-analyticality at this density. When the density approaches the critical value 0.5, the

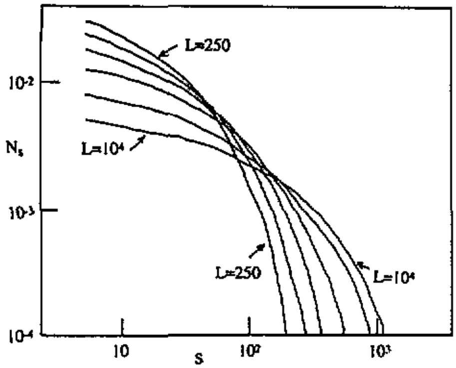

Figure 4. The log–log plot of the cumulative jam-interval distribution N_s against the interval s, for the system size $L = 250$, 500, 1000, 2500, 5000 and 10 000.

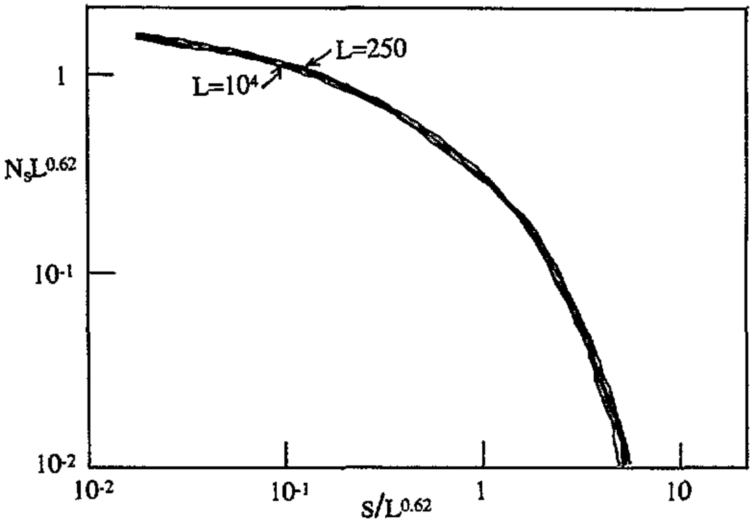

Figure 5. The log–log plot of the rescaled cumulative distribution $N_s L^{0.62}$ against the rescaled interval $sL^{-0.62}$ for the data in figure 4.

self-organized criticality will be maintained even if the density of lane-changing cars is kept constant with increasing L.

We define the typical interval $\langle s \rangle$ of traffic jams as

$$\langle s \rangle \equiv \frac{\sum_{s=1}^{\infty} s^2 n_s}{\sum_{s=1}^{\infty} s n_s} \tag{3}$$

where s is the interval between consecutive jams and n_s is the jam-interval distribution. Figure 3 shows the log–log plot of the jam interval $\langle s \rangle$ against the system size L. The interval $\langle s \rangle$ scales as

$$\langle s \rangle \approx L^{\nu} \qquad \text{with } \nu = 0.62 \pm 0.04. \tag{4}$$

The cumulative interval distribution N_s is defined as

$$N_s \equiv \sum_{s'=s}^{\infty} n_{s'} \tag{5}$$

where n_s is the jam-interval distribution. Figure 4 shows the log–log plot of the cumulative interval distribution N_s against interval s for the system size $L = 250$, 500, 1000, 2500, 5000 and 10 000. We plot the rescaled cumulative interval distribution against the rescaled interval. Figure 5 shows the log–log plot of the rescaled cumulative interval distribution $N_s L^{0.62}$ against the rescaled interval $sL^{-0.62}$ for the data in figure 4. All data collapse onto a single curve. The cumulative interval distribution satisfies the finite-size scaling form

$$N_s(L) \approx L^{-0.62} g(S/L^{0.62}). \tag{6}$$

Therefore, the jam-interval distribution is described by the finite-size scaling form

$$n_s(L) \approx L^{-\beta} f(s/L^{\nu}) \qquad \text{with } \beta = 2\nu \text{ and } \nu = 0.62 \tag{7}$$

where $f(x) = g'(x)$.

We derive the scaling relationship between the exponents β and ν. The total number of jam intervals equals the system size L:

$$L = \sum_{s=1}^{\infty} s n_s L \approx L^{2\nu-\beta+1}. \tag{8}$$

The following scaling relationship is obtained:

$$\beta = 2\nu. \tag{9}$$

Thus, we find that the jam-interval distribution $n_s(L)$ is described by the finite-size scaling (7) with the scaling relation (9). The finite-size scaling form (7) with the scaling relation (9) is consistent with that of the sandpile model with different values for the scaling exponents [21].

In summary, we have presented the 1D asymmetric simple-exclusion model with injection or extraction of particles to investigate the effect of inflow or outflow upon the traffic flow in a highway. We have found that introducing injection or extraction of particles into the asymmetric simple-exclusion model drives the system asymptotically into a steady state exhibiting a self-organized criticality. We have shown that the jam-interval distribution satisfies the finite-size scaling form. We have derived the scaling relation between the scaling exponents.

References

[1] Nagel K and Schreckenberg M 1992 *J. Physique* I **2** 2221
[2] Biham O, Middleton A A and Levine D 1992 *Phys. Rev.* A **46** R6124
[3] Nagatani T 1993 *J. Phys. Soc. Japan* **62** 1085
[4] Nagatani T 1993 *Phys. Rev.* E **48** 3290
[5] Cuesta J A, Martinez F C, Molera J M and Sanchez A 1993 *Phys. Rev.* E R4175
[6] Derrida B, Evans M R, Hakim V and Pasquier V 1993 *J. Phys. A: Math. Gen.* **26** 1493
[7] Gwa L H and Spohn H 1992 *Phys. Rev. Lett.* **68** 725
[8] Liggett T M 1985 *Interacting Particle Systems* (New York: Springer)
[9] Spohn H 1991 *Large Scale Dynamics of Interacting Particles* (Berlin: Springer)
[10] Janowsky S A and Lebowitz J L 1992 *Phys. Rev.* A **45** 618
[11] Bramson M 1988 *J. Stat. Phys.* **51** 863
[12] Meakin P, Ramanlal P, Sander L M and Ball R C 1986 *Phys. Rev.* A **34** 5091
[13] Krug J and Spohn H 1988 *Phys. Rev.* A **38** 4271

[14] Family F and Vicsek T (eds) 1991 *Dynamics of Fractal Surfaces* (Singapore: World Scientific)
[15] Nagatani T 1993 *Physica* **198A** 108
[16] Musha T and Higuchi H 1976 *Japan. J. Appl. Phys.* **15** 1271
[17] Nagatani T 1993 *J. Phys. Soc. Japan* **62** 3837
[18] Ben-Naim E, Krapivsky P L and Redner S 1994 *Phys. Rev.* E **50** 822
[19] Nagel K and Herrmann H J 1993 *Physica* **199A** 254
[20] Bak P, Tang C and Wiesenfeld K 1988 *Phys. Rev.* A **38** 364
[21] Kadanoff L P, Nagel S R, Wu L and Zhou S 1989 *Phys. Rev.* A **39** 6524
[22] Nagel K 1994 *Int. J. Mod. Phys.* C **5** 567